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Fuzzy stochastic damage mechanics (FSDM) based on fuzzy auto-adaptive control theory

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Abstract: In order to fully interpret and describe damage mechanics, the origin and development of fuzzy stochastic damage mechanics were introduced based on the analysis of the harmony of damage, probability, and fuzzy membership in the interval of [0,1]. In a complete normed linear space, it was proven that a generalized damage field can be simulated through β probability distribution. Three kinds of fuzzy behaviors of damage variables were formulated and explained through analysis of the generalized uncertainty of damage variables and the establishment of a fuzzy functional expression. Corresponding fuzzy mapping distributions, namely, the half-depressed distribution, swing distribution, and combined swing distribution, which can simulate varying fuzzy evolution in diverse stochastic damage situations, were set up. Furthermore, through demonstration of the generalized probabilistic characteristics of damage variables, the cumulative distribution function and probability density function of fuzzy stochastic damage variables, which show β probability distribution, were modified according to the expansion principle. The three-dimensional fuzzy stochastic damage mechanical behaviors of the Longtan rolled-concrete dam were examined with the self-developed fuzzy stochastic damage finite element program. The statistical correlation and non-normality of random field parameters were considered comprehensively in the fuzzy stochastic damage model described in this paper. The results show that an initial damage field based on the comprehensive statistical evaluation helps to avoid many difficulties in the establishment of experiments and numerical algorithms for damage mechanics analysis.

Key words: *β probability distribution; fuzzy membership of damage variable; fuzzy auto-adaptive theory; fuzzy stochastic finite element method; fuzzy stochastic damage*

1 Introduction

Nowadays, research on damage mechanics is broadening and deepening with the help of

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uncertain theoretical models originally conceptualized by Zhang and Valliappan (1990a, 1990b), who initiated stochastic damage models. Since then, conventional damage mechanics studies have been further developed based on the probabilistic theory. Based on a micro-mechanics model, damage evolution equations of a solid structure under random loading conditions were set up by Silberschmidt and Chaboche (1994). The damage-rupture development mechanism of discontinuous stochastic composite material reinforced by fibers was analyzed with statistics by Wu and Li (1995). Mechanical characteristics of solid brittle material with plane grain cracks showing correlated random distribution were investigated by Ju and Tseng (1995), who, on the basis of micro-mechanics and the mean-volume theory, introduced a Legedre-Tchebycheff orthogonal polynomial algorithm. With the help of flat noise generator simulation of random factors influencing the medium damage mechanism from both external and internal aspects, continuous damage mechanics were further explored by Silberschmidt (1998), based on his earlier studies. A semi-empirical calculation model, based on the statistical theory of extreme values, was developed by Rinaldi et al. (2007) for statistical analyses of damage simulation. With consideration of hydrochemical effects, Feng et al. (2002) and Qiao et al. (2007) carried out theoretical research on rock damage evolution mechanisms. Finally, some probabilistic conclusions were made by Wang et al. (2006), who applied the damage concepts to rock slope stability analysis.

Dual math coverage implements, including fuzzy-stochastic ones, are still difficult to apply to the engineering domain. Nowadays, most international uncertainty studies on geo-mechanics give more attention to the single math model (Ihara and Tanaka 2000; Dzenis et al. 1993; Dzenis 1996; Bulleit 2004; Wang et al. 2009). Moreover, most uncertain mathematical models use linear coverage that can only simulate irreversible processes from the constitutive model to math coverage. These constitutive models, after the initial simulation of material damage evolution, have to divorce themselves from the uncertain mathematical model and the simulation ends.

It is important to determine uncertain math coverage, i.e., an adaptive mathematical model which, coupled with constitutive functions over a whole time domain for the entire model space, can carry out simulation in accordance with the physical model (e.g., damage mechanics model) all the time. The adaptive mathematical model can identify field distribution of an engineering case to establish objective functional. The output obtained by the functional in generalized space is transformed into a common mechanical field through mathematically certain techniques (e.g., de-fuzzification).

The predominant characteristic of damage measurement is the distance in topological space. The distance interval should be [0,1], in order to guarantee that the damage space is enclosed. The damage in different engineering problems is characterized by fuzzy elasticity (Pierpaolo and Elizabeth 2009; Rigatos and Zhang 2009; Rezaei et al. 2011). With fuzzy elasticity, damage development of the structure and material shows a dynamic nature in its mapping model, which is not taken into account by conventional damage mechanics theory with static mapping definition. Thus, a fuzzy-stochastic damage mechanical model was developed in this study.

2 Fundamentals of fuzzy stochastic damage mechanics (FSDM)

2.1 Randomness and fuzziness of damage variables

Primary concepts of the stochastic damage variable and stochastic damage mechanics were originally established by Zhang and Valliappan (1990a, 1990b, 1998a, 1998b), and the essential hypothesis was verified through Monte-Carlo statistical simulations, based on which the stochastic damage variable shows a β probability distribution. Furthermore, the β probability density function is the only classical probability model whose independent variable spans the interval [0,1], which is in accordance with the characteristics of the damage variable's interval of [0,1] in topological structure and measurement. Moreover, the probability value and fuzzy membership are consistent and both range between [0,1], which enlightens advanced studies on damage mechanics. The original FSDM model is established here based on related work by Zhang et al. (2005).

2.2 Probability distribution of random damage variables

Micro-defects, the cause of material damage, show stochastic distribution. Thus, a damage variable $\varOmega$ also has a stochastic nature and the stochastic damage variable can be established in random space $\varPsi$:

$$\varOmega \in \varLambda_{\alpha'}\ ,\quad \varLambda_{\alpha'} = \left\{ \boldsymbol{U}_{\alpha'}, \boldsymbol{e}_{\alpha'}, \boldsymbol{\sigma}_{\alpha'}, \boldsymbol{f}_{\alpha'} \right\},$$
$$\bigcup_{\alpha' \in \boldsymbol{X}} \varLambda_{\alpha'} \subset \varPsi \quad \left(\alpha' \in \boldsymbol{X} = \left(x_1, x_2, \cdots, x_n \right)^{\mathrm{T}} \right) \tag{1}$$

where α' is a stochastic subset of a stochastic vector $\boldsymbol{X} = \left(x_1, x_2, \cdots, x_n \right)^{\mathrm{T}}$, and $\varLambda_{\alpha'}$ is a probabilistic set derived from α', consisting of a probabilistic nodal displacement vector $\boldsymbol{U}_{\alpha'}$, probabilistic body force vector $\boldsymbol{f}_{\alpha'}$, stochastic stress tensor $\boldsymbol{\sigma}_{\alpha'}$, and stochastic strain tensor $\boldsymbol{e}_{\alpha'}$.

The crucial propositions are as follows:

Proposition 1: $\varPsi_1$ is defined as a probability space of independent β distribution, and [0,1] is defined as the domain of a stochastic vector $\boldsymbol{x}$ in $\varPsi_1$; that is, $\boldsymbol{x} \sim \beta\left(p,q \right) : \varPsi_1 \left| \boldsymbol{x} = [0,1] \right.$, where $\beta\left(p,q \right)$ is the β distribution function, and p and q are the parameters for the β distribution function. Therefore, $\varPsi_1$ is a Banach space under the ∞ norm, or a complete normed linear space. The proof of this proposition was described by Wang and Zhang (2012).

Meanwhile, the β probabilistic cumulative distribution function vector can be defined as $\boldsymbol{y}_1$. $\varPsi_2$ is defined as a probability space of the damage variable $\varOmega$, which shows independent B distribution over [0,1], i.e., $\varOmega \sim B : \varPsi_2 \left| \varOmega = [0,1] \right.$. Following the same rule, it can be proven under the ∞ norm that $\varPsi_2$ is a Banach space, which can be expressed as $\varOmega \sim B : \varPsi_2 \left| \varOmega = [0,1] \right.$. Meanwhile, the B probabilistic cumulative distribution function vector can be defined as $\boldsymbol{y}_2$.

Proposition 2: The necessary and sufficient condition for coincidence of the probability spaces Ψ_1 and Ψ_2 is that vector $\boldsymbol{y}_2$ converges to vector $\boldsymbol{y}_1$ with the same definition domain [0,1] under the ∞ norm. The proposition has been proved by Wang and Zhang (2012).

Based on this proposition, β probability distribution can be used to simulate B probability distribution of damage variable Ω, and this procedure can be applied to engineering through the law of averages over [0,1].

2.3 Fuzzy membership constitution of damage variable

Mechanical definition of damage is the macro-effect produced by micro-crack expansion as well as evolution and the development of material deformation. The damage is a physical-mechanical process from micro-defect to macro-behavior (the safety status of working conditions). A quantitative index ϖ that can quantify the material's micro-defect is called a damage measuring index (DMI). Γ is a fuzzy analytical domain for ϖ, i.e., $\varpi \in \Gamma$. Γ must be defined in a fuzzy space $\Xi\left(\tilde{C}, \tilde{L}, \tilde{P}\right)$, where $\tilde{C}$, $\tilde{L}$, and $\tilde{P}$ are the fuzziness of physical parameters (including Ω), loading conditions, and constraint conditions, respectively. The key technological question for damage simulation is which scale of ϖ means the material's damage or how ϖ rates damage. The fuzzy mechanism of macro-working behavior has been discussed in many studies. Micro-defects, however, inevitably induce macro-deformation, and this essential mechanism describes the fuzziness of ϖ evolution. Ω, as a fuzzy functional over DMI, represents the magnitude of the membership value of ϖ in domain Γ, and can analyze damage variable development as in Eq. (2):

$$\Omega = \mu_{\varpi \in \Gamma}\left[\omega(\varpi)\right] = \omega_{\Gamma}(\varpi) \tag{2}$$

where $\mu_{\varpi \in \Gamma}$ is a fuzzy membership function, namely, $\varpi \in \Gamma : \tilde{A} \subset \Xi\left(\tilde{C}, \tilde{L}, \tilde{P}\right) \big| \Omega \in [0,1]$; $\tilde{A}$ is the fuzzy subset in domain Γ and includes three kinds of fuzziness, represented by $\tilde{C}$, $\tilde{L}$, and $\tilde{P}$; ω is the probabilistic integral of ϖ; and ω_{Γ} is the function of the generalized probabilistic integral variable.

The key for damage simulation is how to establish the ϖ model and fuzzy functional Ω model. After that, ω and ω_{Γ} can be obtained through probabilistic integration. As for most geo-material with elasto-plastic constitution, a structure's damage is the result of volumetric deformation and deviation deformation. Based on these facts, this study defined DMI as follows:

$$\varpi = \frac{\left|\sigma_{\mathrm{m}}\right| \tan\varphi + c}{\left|J_3\right|^{1/3}} \tag{3}$$

where φ and c are the internal friction angle and cohesive stress, respectively, and σ_{m} and J_3 are the hydrostatic pressure and the third invariable of the deviation stress tensor, respectively (Qian 1980). ϖ represents the ratio of the volumetric deformation ($\left|\sigma_{\mathrm{m}}\right| \tan\varphi + c$) to the deviation deformation ($\left|J_3\right|^{1/3}$).

Three cases were considered in the fuzzification process for ϖ-forming fuzzy functional memberships: (1) the deviation deformation is superior to volumetric deformation in magnitude when material damage is developing; (2) when ϖ value reaches 0.5 (gray space of the fuzzy domain), the damage evolution can be revealed by the functional distribution figure. The damage evolution has a decreasing tendency during the early stage due to the neutralization effect, and the increase in the volumetric deformation during the later stage causes damage accumulation; and (3) the damage evolution is almost consistent with the second case, and the primary characteristics are that gray space can be established from decision analysis based on the measured data during the de-fuzzification process for a fuzzy output, and thereby the result of magnitude comparison of two kinds of deformation is always effective over the whole fuzzy span [0,1]. For the three cases, the corresponding fuzzy functional memberships were established: half-depressed distribution (Eq. (4) and Fig. 1(a)), swing distribution (Eq. (5) and Fig. 1(b)), and combined swing distribution (Eq. (6) and Fig. 1(c)):

$$\Omega = \begin{cases} 1 & 0 < \varpi \le 0.6 \\ \dfrac{1.5-\varpi}{1.5-0.6} & 0.6 < \varpi \le 1.5 \\ 0 & \varpi > 1.5 \end{cases} \tag{4}$$

$$\Omega = \begin{cases} 1 & 0 < \varpi \le 0.44 \\ 1.5-\mathrm{e}^{-(\ln\varpi)^2} & 0.44 < \varpi \le 2.33 \\ 1 & \varpi > 2.33 \end{cases} \tag{5}$$

$$\Omega = \begin{cases} 1 & 0 < \varpi \le 0.44 \\ 1.5-\mathrm{e}^{-(\ln\varpi)^2} & 0.44 < \varpi \le 1.0 \\ 1.5-\varpi & 1.0 < \varpi \le 1.5 \\ 0.996-\mathrm{e}^{-2\left[\ln(\varpi-0.45)\right]^2} & \varpi > 1.5 \end{cases} \tag{6}$$

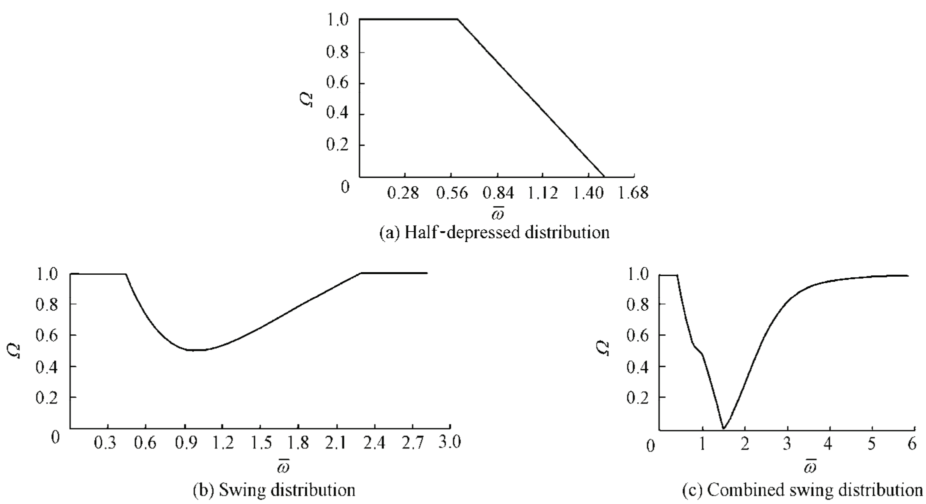

Fig. 1 Fuzzy functional memberships

2.4 Generalized damage variable under double mathematics coverage

When randomness and fuzziness simultaneously exist in the structure damage evolution process, according to the expansion principle, the single fuzzy space $\Xi\left(\tilde{C},\tilde{L},\tilde{P}\right)$ and stochastic space $\Psi:\bigcup_{\alpha\in \boldsymbol{X}}\Lambda_{\alpha}\subset\Psi\ \left(\alpha\in \boldsymbol{X}=\left(x_1,x_2,\cdots,x_n\right)^{\mathrm{T}}\right)$ of the damage variable need to be expanded to a generalized uncertain space $O:\xi\left(s,f\right)$, $\boldsymbol{U}_{s,f},\boldsymbol{e}_{s,f},\boldsymbol{\sigma}_{s,f},\boldsymbol{f}_{s,f}\subset O:\xi\left(s,f\right)$, where s and f are the stochastic coverage and fuzzy coverage, respectively; $\xi\left(s,f\right)$ is the sub-group of generalized uncertain space; $\boldsymbol{U}_{s,f}$ is a global fuzzy-stochastic displacement column matrix; $\boldsymbol{e}_{s,f}$ is a generalized uncertain strain tensor; $\boldsymbol{\sigma}_{s,f}$ is a fuzzy-stochastic stress tensor; and $\boldsymbol{f}_{s,f}$ is a generalized body force vector (Wang et al. 2007).

The cumulative distribution function (CDF) and probability density function (PDF) obtained by the expansion principle and fuzzy-probabilistic integration are functions of fuzzy functional memberships of the generalized damage variable Ω. Taking the combined swing distribution as an example, the generalized CDF and PDF for the classical damage fuzzy set can be established based on the definition of DMI:

$$F_{\Omega}^{f}\left(\Omega\right)=\frac{1}{B\left(p,q\right)}\int_0^{\Omega}\omega^{p-1}\left(1-\omega\right)^{q-1}\mu_{\varpi\in\Gamma}\left[\omega\left(\varpi\right)\right]\mathrm{d}\omega=\begin{cases}\dfrac{1}{B\left(p,q\right)}\displaystyle\int_0^{\Omega}\omega^{p-1}\left(1-\omega\right)^{q-1}\cdot 1\mathrm{d}\omega & 0<\varpi\leq 0.44\\ \dfrac{1}{B\left(p,q\right)}\displaystyle\int_0^{\Omega}\omega^{p-1}\left(1-\omega\right)^{q-1}\left[1.5-\mathrm{e}^{-\left(\ln\varpi\right)^2}\right]\mathrm{d}\omega & 0.44<\varpi\leq 1.0\\ \dfrac{1}{B\left(p,q\right)}\displaystyle\int_0^{\Omega}\omega^{p-1}\left(1-\omega\right)^{q-1}\left(1.5-\varpi\right)\mathrm{d}\omega & 1.0<\varpi\leq 1.5\\ \dfrac{1}{B\left(p,q\right)}\displaystyle\int_0^{\Omega}\omega^{p-1}\left(1-\omega\right)^{q-1}\left[0.996-\mathrm{e}^{-2\left[\ln\left(\varpi-0.45\right)\right]^2}\right]\mathrm{d}\omega & \varpi>1.5\end{cases}\tag{7}$$

$$f_{\Omega}^{f}\left(\Omega\right)=\frac{1}{B\left(p,q\right)}\Omega^{p-1}\left(1-\Omega\right)^{q-1}\mu_{\varpi\in\Gamma}\left[\omega\left(\varpi\right)\right]=\begin{cases}\dfrac{1}{B\left(p,q\right)}\Omega^{p-1}\left(1-\Omega\right)^{q-1}\cdot 1 & 0<\varpi\leq 0.44\\ \dfrac{1}{B\left(p,q\right)}\Omega^{p-1}\left(1-\Omega\right)^{q-1}\left[1.5-\mathrm{e}^{-\left(\ln\varpi\right)^2}\right] & 0.44<\varpi\leq 1.0\\ \dfrac{1}{B\left(p,q\right)}\Omega^{p-1}\left(1-\Omega\right)^{q-1}\left(1.5-\varpi\right) & 1.0<\varpi\leq 1.5\\ \dfrac{1}{B\left(p,q\right)}\Omega^{p-1}\left(1-\Omega\right)^{q-1}\left[0.996-\mathrm{e}^{-2\left[\ln\left(\varpi-0.45\right)\right]^2}\right] & \varpi>1.5\end{cases}\tag{8}$$

With these governing functions, the fuzzy stochastic damage reliability (FSDR) can be computed using the equivalent-normal differential checking-point theorem (Wang 2004; Wang et al. 2008; Zhu 1993).

3 Fuzzy stochastic damage finite element method

The key methodology for FSDM realization is the establishment of the fuzzy stochastic damage finite element method (FSD-FEM). The FSD constitution model was assimilated into FSD-FEM. The constitutional component for the methodology is the damage function gradient $\nabla g_{\alpha*}$, which can be expressed as follows:

$$\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)=\boldsymbol{T}^{-1}\nabla g_{\alpha*}\left(\boldsymbol{Y}\right)=\left(\partial g_{\alpha*}/\partial Y_1^*,\partial g_{\alpha*}/\partial Y_2^*,\cdots,\partial g_{\alpha*}/\partial Y_n^*\right)^{\mathrm{T}} \tag{9}$$

where $\boldsymbol{Y}^*$ is the independent standard normal vector with its element Y_i^* $\left(i = 1, 2, \cdots, n\right)$; $\boldsymbol{T}^{-1}$ is the inverse matrix of $\boldsymbol{T}$, which is a diagonal matrix of stochastic characteristics; and $\boldsymbol{Y}$ is the independent non-standard normal vector.

Based on the studies in Section 2.4, the checking-point iterative direction, namely, the unit vector in the negative gradient direction $\boldsymbol{\alpha}$, can be computed by Eq. (10):

$$\boldsymbol{\alpha}=\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)\Big/\left\|\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)\right\| \tag{10}$$

where $\|*\|$ is a Euclidean norm operator. $\left\|\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)\right\|$ can be computed by Eq. (11):

$$\left\|\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)\right\|=\sqrt{\left(\frac{\partial g_{\alpha*}}{\partial Y_1^*}\right)^2+\left(\frac{\partial g_{\alpha*}}{\partial Y_2^*}\right)^2+\cdots+\left(\frac{\partial g_{\alpha*}}{\partial Y_n^*}\right)^2} \tag{11}$$

$\boldsymbol{\alpha}$ is the direction cosine of the reliability index along axis Y_i^*. Thus, $\boldsymbol{\alpha}$ is perpendicular to the ultimate status surface against the coordinate system origin. $g_{\alpha*}\left(\boldsymbol{Y}^*\right)$ will descend the fastest when the checking-point is computed iteratively along this directional cosine.

Then, the iterative step size of the checking-point, d, can be determined by Eq. (12):

$$d = g_{\alpha*}\left(\boldsymbol{Y}^*\right)\Big/\left\|\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)\right\| \tag{12}$$

In order to ensure the line connecting the origin of the kth iterative $\left(\boldsymbol{Y}^*\right)^k$ and the new iterative coordinate $\left(\boldsymbol{Y}^*\right)^{k+1}$ along the gradient direction of the curve, on which $\left(\boldsymbol{Y}^*\right)^k$ is distributed, modification of $\left(\boldsymbol{Y}^*\right)^k$ is the crucial technology for this algorithm and can be expressed as

$$\left(\boldsymbol{Y}^*\right)^{k'}=\left[\left(\boldsymbol{Y}^*\right)^k\right]^{\mathrm{T}}\boldsymbol{\alpha\alpha} \tag{13}$$

where $\left(\boldsymbol{Y}^*\right)^{k'}$ is the modified $\left(\boldsymbol{Y}^*\right)^k$.

Therefore, the controlling iterative function can be deduced as follows:

$$\left(\boldsymbol{Y}^*\right)^{k+1}=\left\{\left[\left(\boldsymbol{Y}^*\right)^k\right]^{\mathrm{T}}\boldsymbol{\alpha}+\frac{g_{\alpha*}\left(\boldsymbol{Y}^*\right)}{\left\|\nabla g_{\alpha*}\left(\boldsymbol{Y}^*\right)\right\|}\right\}\boldsymbol{\alpha} \tag{14}$$

Then, the updated checking-point vector $\boldsymbol{Y}^{k+1}$ and the status vector of object system $\left(\boldsymbol{X}^*\right)^{k+1}$ for the numerical back-substitution algorithm are established:

$$\boldsymbol{Y}^{k+1}=\boldsymbol{T}^{-1}\left[\left(\boldsymbol{Y}^*\right)^{k+1}-\boldsymbol{T}\right],\ \left(\boldsymbol{X}^*\right)^{k+1}=\left(\boldsymbol{A}^{\mathrm{T}}\right)^{-1}\boldsymbol{Y}^{k+1} \tag{15}$$

Based on Eq. (9), the damage reliability index β^*, and damage failure probability P_{f}^* can be calculated eventually with Eq. (16):

$$\beta^* = \sqrt{\left(\boldsymbol{Y}^*\right)^{\mathrm{T}} \boldsymbol{Y}^*} = \sqrt{\left(Y_1^*\right)^2 + \left(Y_2^*\right)^2 + \cdots + \left(Y_n^*\right)^2}, \quad P_{\mathrm{f}}^* = 1 - \Phi\left(\beta^*\right) \tag{16}$$

where Φ is the cumulative distribution function of standard normal distribution.

The three-dimensional fuzzy stochastic damage (3DFSD) computation program was developed in a Digital Visual Fortran workspace.

4 Verification and application to engineering project

The Longtan Dam is one of the great hydraulic rolled-concrete structures in China. Most zones of the dam body are composed of rolled concrete (Wang and Zhang 2008). The numerical model and material zoning are shown in Fig. 2 and Fig. 3.

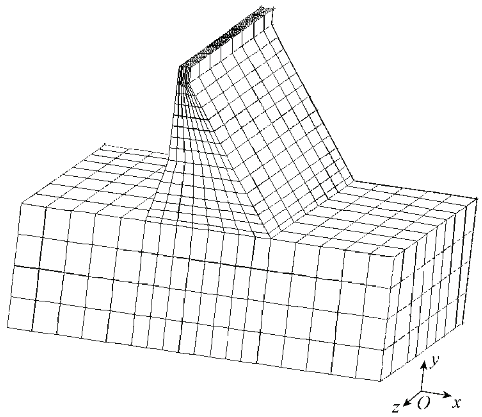

Fig. 2 Numerical model of Longtan rolled-concrete dam

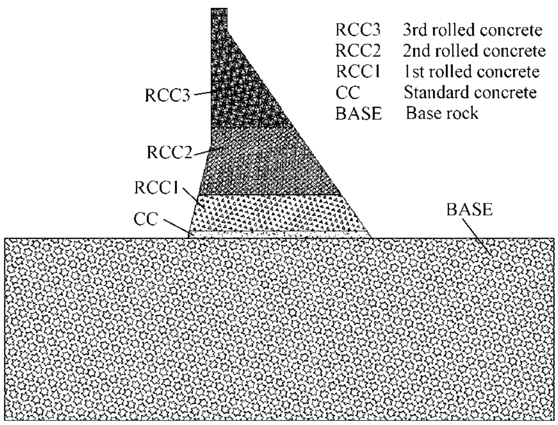

Fig. 3 Material zoning of Longtan rolled-concrete dam

The four-parameter failure criterion was used for the rolled-concrete dam to simulate damage development of materials (Wang et al. 2011). The four primary parameters during simulation were constant: A = 2.010 8, B = 0.971 4, C = 9.141 2, and D = 0.231 2. This study took into account six random parameters: the Young's modulus, Poisson ratio, cohesive stress, friction angle, bulk specific gravity, and ultimate compressive strength. The expectation values and variation values of the six random parameters of diverse materials are shown in Table 1 and

Table 2. There remains one fuzzy-stochastic parameter, the statistic independent index, i.e., damage variable Ω.

Table 1 Expected values of material parameters of Longtan rolled-concrete dam

Material code	Expected value					
	Young's modulus (GPa)	Poisson ratio	Cohesive stress (MPa)	Friction angle (°)	Bulk specific gravity (kN/m^3)	Ultimate compressive strength (MPa)
RCC3	20	0.167	4.90	0.90	24.0	20
RCC2	20	0.167	5.49	0.91	24.0	20
RCC1	20	0.167	7.54	0.94	24.0	20
CC	21	0.250	8.93	0.94	24.5	21
BASE	16	0.300	1.63	0.88	25.0	19

Table 2 Variation of material parameters of Longtan rolled-concrete dam

Material code	Variation					
	Young's modulus	Poisson ratio	Cohesive stress	Friction angle	Bulk specific gravity	Ultimate compressive strength
RCC3	0.11	0.10	0.07	0.06	0.06	0.09
RCC2	0.10	0.10	0.08	0.09	0.07	0.08
RCC1	0.10	0.10	0.08	0.09	0.07	0.08
CC	0.10	0.05	0.09	0.08	0.10	0.09
BASE	0.08	0.08	0.08	0.06	0.08	0.08

The development and distribution of a generalized damage field in the Longtan rolled-concrete gravity dam, under gravity conditions, were studied with FSD-FEM.

Figs. 4 through 6 show the displacement contours of the dam without damage development under gravity conditions. Concrete structures, due to their rigid plastic mechanisms, can easily fail at discontinuous places or transition locations where stress concentration and corresponding damage development occur. Contour distributions of the displacement field and strain field can uncover these failure conditions of structures.

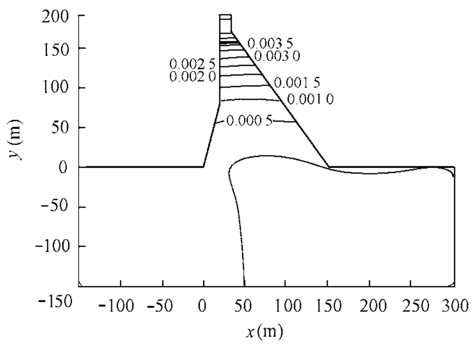

Fig. 4 Contour of *x*-displacement at cross-section before damage (unit: m)

The concentration of displacement growth occurs more at the dam top than in other places. The maximum magnitude of displacement in the *x* direction stays at the dam top where the displacement level reaches 10^{-2} m. Owing to the high compressive strength of concrete, the displacement in the *y* direction shows gradual diffusion from top to bottom through the whole

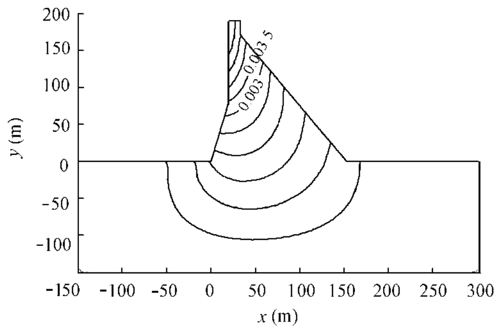

Fig. 5 Contour of *y*-displacement at cross-section before damage (unit: m)

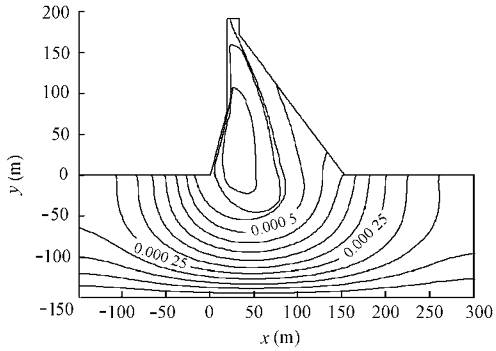

Fig. 6 Contour of *z*-displacement at cross-section before damage (unit: m)

dam body. The dam abutment is generally not an area of the gravity dam vulnerable to failure. Thus, the magnitude of displacement in the z direction is not high. It reaches 10^{-3} m. The characteristics described above are in agreement with the objective phenomena, which show that the results of the 3DFSD program are reliable for engineering application.

According to Fig. 7 and Fig. 8, the expected values of the damage variable diminished evidently after the random parameters were normalized equivalently. Meanwhile, the expected value of the damage variable increased significantly at some sites, including the connecting segment of the dam crest, the upstream dam ankle, the downstream dam toe, and the connecting segments of the dam foundation. These characteristics have been described in previous studies for rigid-plastic concrete structures with discontinuous outlines, where loading accumulated and concentrated (Liu 2007). The studies show that the stress level at these discontinuous sites increases quickly, by which it can be proven that the material is inclined to fracture there and these zones experience abundant damage.

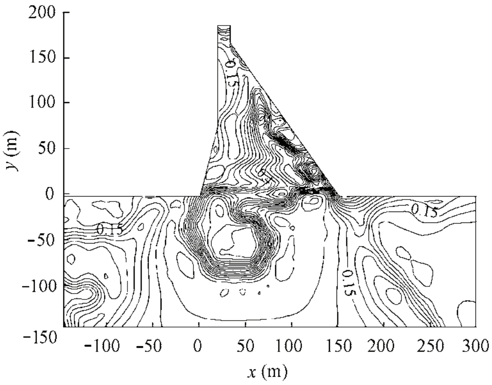

Fig. 7 Contour of expectation of damage variable at cross-section before equivalent normalization

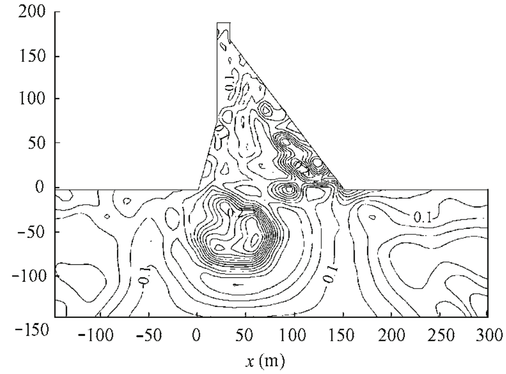

Fig. 8 Contour of expectation of damage variable at cross-section after equivalent normalization

Fig. 9 shows that the level of the mean square deviation of the damage variable is high at the upstream dam ankle, the downstream dam toe, and the connecting segments of the dam foundation, and these zones tend to converge densely. These facts demonstrate that materials of these zones are vulnerable to yield and fracture due to the development of a generalized damage field (Wang et al. 2011). Based on previous studies (Zhang and Cai 2010), the media would be simulated as macro-homogeneous while the crack evolution showed wide variation during damage field development. It is vitally important, however, that the generalized damage field is heterogeneous (Wang and Zhang 2010).

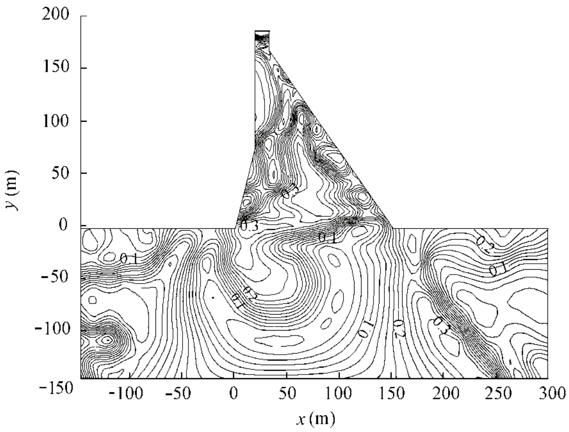

Fig. 9 Contour of mean square deviation of damage variable at cross-section after equivalent normalization

According to Fig. 10, the level of the reliability index at the sites with concentrated damage development decreases, showing that the failure probability of material at these sites is high (Wang and Zhang 2008, 2009a, 2009b).

Fig. 11 shows that the variance of σ_x reaches 20 kPa^2 at dam ankle zones and dam toe zones, where σ_x is the component of generalized stress tensor in the *x* direction of principal axes. Thus, the degree of dispersion of the stress level was significant at these sites and the corresponding safety degree was inclined to be out of control (Qiu et al. 2004).

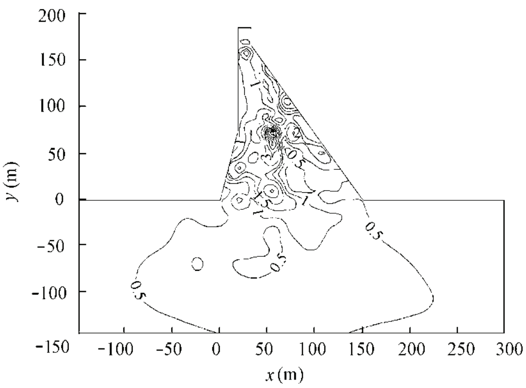

Fig. 10 Contour of β^* at cross-section after equivalent normalization

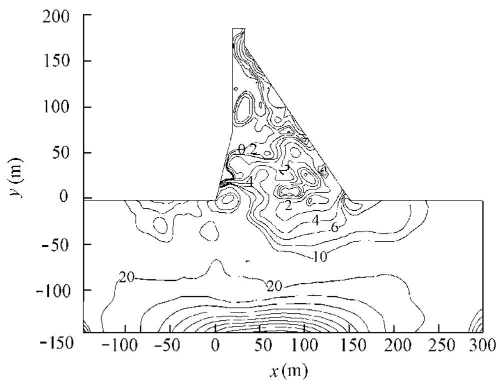

Fig. 11 Contour of variance of σ_x at cross-section equivalent normalization (unit: kPa^2)

5 Conclusions

(1) With some primary concepts of FSDM, three primary distributions of the fuzzy

stochastic damage variable, the half-depressed distribution, swing distribution, and combined swing distribution, were developed based on fuzzy functional memberships.

(2) The numerical method, FSD-FEM, was developed and applied to the Longtan rolled-concrete dam. The primary output fields, i.e., displacement, stress, damage variable, and damage reliability index, were examined through spatial distribution of their statistical characteristics, and the results conformed to those from previous research.

(3) Crucial characteristics of FSDM such as statistical correlation, non-normal distribution, and fuzzy extensionality were assimilated into FSD-FEM. The uncertainty of damage variable was improved, and two primary uncertain characteristics, fuzziness and randomness of damage, were incorporated into the fuzzy stochastic damage mechanics theorem.

References

Bulleit, W. M. 2004. Stochastic damage models for wood structural elements. *Proceedings of the 2001 Structures Congress and Exposition*. Washington, D.C.: ASCE. [doi:10.1061/40558(2001)183]

Dzenis, Y. A., Bogdanovich, A. E., and Pastore, C. M. 1993. Stochastic damage evolution in textile laminates. *Composites Manufacturing*, 4(4), 187-193. [doi:10.1016/0956-7143(93)90003-Q]

Dzenis, Y. A. 1996. Stochastic damage evolution modeling in laminates. *Journal of Thermoplastic Composite Materials*, 9(1), 21-34. [doi:10.1177/089270579600900103]

Feng, X. T., Wang, C. Y., and Chen, S. L. 2002. Testing study and real-time observation of rock meso-cracking process under chemical erosion. *Chinese Journal of Rock Mechanics and Engineering*, 21(7), 935-939. (in Chinese)

Ihara, C., and Tanaka, T. 2000. A stochastic damage accumulation model for crack initiation in high-cycle fatigue. *Fatigue and Fracture of Engineering Materials and Structures*, 23(5), 375-380. [doi:10.1046/j.1460-2695.2000.00308.x]

Ju, J. W., and Tseng, K. H. 1995. An improved two-dimensional micromechanical theory for brittle solids with randomly located interacting microcracks. *International Journal of Damage Mechanics*, 4(1), 23-57. [doi:10.1177/105678959500400103]

Liu, Y. W. 2007. Improving the abrasion resistance of hydraulic-concrete containing surface crack by adding silica fume. *Construction and Building Materials*, 21(5), 972-977. [doi:10.1016/j.conbuildmat.2006.03.001]

Pierpaolo, D. U., and Elizabeth, A. M. 2009. Autocorrelation-based fuzzy clustering of time series. *Fuzzy Sets and Systems*, 160(24), 3565-3589. [doi:10.1016/j.fss.2009.04.013]

Qian, W. C. 1980. *Variation Method and Finite Element Method*. Beijing: Scientific Press. (in Chinese)

Qiao, L. P., Liu, J., and Feng, X. T. 2007. Study on damage mechanism of sandstone under hydro-physico-chemical effects. *Chinese Journal of Rock Mechanics and Engineering*, 26(10), 2117-2124. (in Chinese)

Qiu, Z. H., Zhang, W. H., and Ren, T. H. 2004. Nonlinear dynamic damage analysis of dam and rock foundation under the action of earthquake. *Chinese Journal of Rock Mechanics and Hydraulic Engineering*, 36(5), 629-636. (in Chinese)

Rezaei, M., Monjezi, M., and Varjani, A. Y. 2011. Development of a fuzzy model to predict flyrock in surface mining. *Safety Science*, 49(2), 298-305. [doi:10.1016/j.ssci.2010.09.004]

Rigatos, G., and Zhang, Q. 2009. Fuzzy model validation using the local statistical approach. *Fuzzy Sets and Systems*, 160(7), 882-904. [doi:10.1016/j.fss.2008.07.008]

Rinaldi, A., Krajcinovic, D., and Mastilovic, S. 2007. Statistical damage mechanics and extreme value theory. *International Journal of Damage Mechanics*, 16(1), 57-76. [doi:10.1177/1056789507060779]

Silberschmidt, V. V., and Chaboche, J. L. 1994. Effect of stochasticity on the damage accumulation in solids. *International Journal of Damage Mechanics*, 3(1), 57-70. [doi:10.1177/105678959400300103]

Silberschmidt, V. V. 1998. Dynamics of stochastic damage evolution. *International Journal of Damage Mechanics*, 7(1), 84-98. [doi:10.1177/105678959800700104]

Wang, J. C., Chang, L. S., and Chen, Y. J. 2006. Study on probability damage evolutionary rule of joined rock mass slope. *Chinese Journal of Rock Mechanics and Engineering*, 25(7), 1396-1401. (in Chinese)

Wang, Y. J. 2004. *Fuzzy-random Theory Application on Geo-engineering Uncertain Analysis*. M. E. Dissertation. Wuhan: Yangtze River Scientific Research Institute. (in Chinese)

Wang, Y. J., Zhang, W. H., and Jin, W. L. 2007. Stochastic finite element analysis for fuzzy probability of embankment system failure by first-order approximation theorem. *Journal of Zhejiang University* (*Engineering Science*), 41(1), 52-56. (in Chinese)

Wang, Y. J., and Zhang, W. H. 2008. Research on fuzzy self-adapting finite element in stochastic damage mechanics analysis for Longtan rolled-concrete dam. *Chinese Journal of Rock Mechanics and Engineering*, 27(6), 1251-1259. (in Chinese)

Wang, Y. J., Zhang, W. H., Jin, W. L., Wu, C. Y., and Ren, D. C. 2008. Fuzzy stochastic generalized reliability studies on embankment systems based on first-order approximation theorem. *Water Science and Engineering*, 1(4), 36-46. [doi:10.3882/j.issn.1674-2370.2008.04.004]

Wang, Y. J., and Zhang, W. H. 2009a. Rock slope reliability studies based on fuzzy stochastic damage theory. *Journal of Shengyang Jianzhu University* (*Natural Science*), 25(3), 421-425. (in Chinese)

Wang, Y. J., and Zhang, W. H. 2009b. Fuzzy self-adapting stochastic damage mechanism for Jingnan main dike of Yangtse Rive. *Journal of Zhejiang University* (*Engineering Science*), 43(4), 743-749, 776. (in Chinese) [doi:10.3785/j.issn.1008-973X.2009.04.026]

Wang, Y. J., Zhang, W. H., Wu, C. Y., and Ren, D. C. 2009. Three-dimensional stochastic seepage field for embankment engineering. *Water Science and Engineering*, 2(1), 58-71. [doi:10.3882/j.issn.1674-2370.2009.01.006]

Wang, Y. J., and Zhang, W. H. 2010. Studies on non-linear fuzzy stochastic damage field. *Journal of Hydraulic Engineering*, 41(2), 189-197. (in Chinese)

Wang, Y. J., Zhang, W. H., Zhang, C. H., and Jin, F. 2011. Generalized damage reliability and sensitivity analysis on rolled-concrete gravity dam. *Journal of Civil, Architectural and Environmental Engineering*, 33(1), 77-86. (in Chinese)

Wang, Y. J., and Zhang, W. H. 2012. Super gravity dam generalized damage study. *Advanced Materials Research*, 479-481, 421-425. [doi:10.4028/www.scientific.net/AMR.479-481.421]

Wu, H. C., and Li, V. C. 1995. Stochastic process of multiple cracking in discontinuous random fiber reinforced brittle matrix composites. *International Journal of Damage Mechanics*, 4(1), 83-102. [doi:10.1177/105678959500400105]

Zhang, W. H., and Valliappan, S. 1990a. Analysis of random anisotropic damage mechanics problems of rock mass, part I: Probabilistic simulation. *Rock Mechanics and Rock Engineering*, 23(2), 91-112. [doi:10.1007/BF01020395]

Zhang, W. H., and Valliappan, S. 1990b. Analysis of random anisotropic damage mechanics problems of rock mass, part II: Statistical estimation. *Rock Mechanics and Rock Engineering*, 23(4), 241-259. [doi:10.1007/BF01043306]

Zhang, W. H., and Valliappan, S. 1998a. Continuum damage mechanics theory and application, part I: Theory. *International Journal of Damage Mechanics*, 7(3), 250-273. [doi:10.1177/105678959800700303]

Zhang, W. H., and Valliappan, S. 1998b. Continuum damage mechanics theory and application, part II: Application. *International Journal of Damage Mechanics*, 7(3), 274-297. [doi:10.1177/105678959800700304]

Zhang, W. H., Jin, W. L., and Li, H. B. 2005. Stability analysis of rock slope based on random damage mechanics. *Journal of Hydraulic Engineering*, 36(4), 413-419. (in Chinese)

Zhang, W. H., and Cai, Y. Q. 2010. *Continuum Damage Mechanics and Numerical Applications (Advanced Topics in Science and Technology in China)*. Hangzhou: Zhejiang University Press. (in Chinese)

Zhu, Y. X. 1993. *Slope Reliability Analysis*. Beijing: China Metallurgical Industry Press. (in Chinese)

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